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ABSTRACT

By varying task conditions and constraints on subjects, this study attempted to extend and refine a developmental theory of partitioning as a foundational process undergirding children's construction of a rational number. Subjects were 31 preponderantly higher-grade pupils enrolled at an elementary school in Nova Scotia, Canada. Two tasks were used in the experiment. The first, taken directly from an earlier study and used to introduce and familiarize the child with the clinical setting, involved the partitioning of circular and rectangular "birthday cakes." The second, designed specifically for the present study, involved the partitioning of "giant cookies" in regular triangular and regular pentagonal shapes. This latter procedure allowed subjects to choose the number of pieces into which they could partition the shapes before they actually did so. Results indicated that children think it is easier to partition into two or four parts than into three or five parts, even when the shapes of the objects to be partitioned are as much or more suited to partitioning into three or five parts. In addition, children experienced much more success when partitioning into two and four parts than they did when partitioning into three or five. (RH)

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Children's Judgements Concerning "Easy" Partitioning Tasks as Related to a Theory of Partitioning

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Introduction

In an earlier study (Pothier & Sawada, 1983), a five-level theoretical account of the development in young children of partitioning as a foundational process undergirding the child's construction of rational number was proposed. The general purpose of the present study was to further extend and refine the theory by (1) examining the theory for invariance when (a) other geometrical shapes are involved in the partitioning tasks, and (b) when children are given free choice in deciding how many partitions into which a given object would be easily partitioned, (2) assessing some auxiliary hypotheses (relating to the sequence of the levels) which were assumed in conducting the original study. In that study, the tasks used with children were characterized as follows: (1) as concerns shape, the objects were either circular or rectangular, (2) as concerns procedure, the number of equal parts the child was asked to partition the objects (a "cake" or a "cookie") into was always specified by the researcher, and (3) as concerns sequence, the order of tasks performed by the children was always the same in that each child dealt with a task that required partitioning into two or four equal shares before having to partition an object into three or five equal shares. The present study was designed to extend the generality of these conditions by developing a new task which, as concerns shape, involved triangular and pentagonal "cookies" for which children, as concerns procedure and sequence, had freedom to choose the number of pieces into which they could partition the cookies before they actually partitioned them.

Rationale

Since the study was planned to simultaneously assess all three conditions, it is important to briefly discuss each condition and the relationships among them. In reference to the third condition (and as suggested in the earlier study), it is possible that in solving tasks involving an even number of partitions, techniques leading to success may induce a mind set which interferes with subsequent performance on tasks requiring an odd number of partitions. This possibly becomes a real concern when it is realized that in the earlier study the partitioning procedures used by children for halving and quartering tended to persist even in contexts for which the were not particularly appropriate. Furthermore, since the difference between Level III (the Evenness Level) and Level IV (the Oddness Level) is predicated on the earlier mastery of

partitioning into an even as compared with an odd number of shares the possible existence of this sequence effect is theoretically unsatisfactory. In testing this effect, the shape of the objects involved in the partitioning tasks is critical in that some shapes are more compatible with partitioning into an even number of shares while others are more compatible with an odd partitioning. For example, whereas a rectangular shape may be neutral in its compatibility, there is a sense of threeness in triangular shapes and a sense of fiveness in pentagonal shapes which might serve to trigger the use of partitioning strategies to obtain odd numbers of shares if indeed children are capable of using them. Moreover, if:

1. children are given a free choice in deciding the number of parts into which a triangular or pentagonal object would be easily partitioned, and if

2. children still chose to select partitioning in 2 or 4, and if,

3. children experienced more success in partitioning in 2 or 4 parts than when partitioning in 3 or 5,

then the earlier finding that even partitioning precedes odd (for the range of numbers involved) would be supported. At the same time, the use of these "odd" shapes provides an extension of the generalizability of the original theory, at Level II and Level III, over these new shapes; and as well, the provision of free choice generates data on the question of children's awareness of the compatibility between particular partitioning procedures and the shape of the objects being partitioned.

Tasks

Two tasks, both presented in the context of a birthday party with miniature dolls (9 cm tall) representing the number of children at the party, were used. The first task, taken directly from the earlier study, involved the partitioning of circular and rectangular birthday cakes, and was used to introduce and familiarize the child with the clinical setting. The second task, designed specifically for the present study, involved the partitioning of "giant cookies", in regular triangular and regular pentagonal shape, each cut from coloured art foam and mounted on a stiff cardboard background. Sticks used to indicate cuts on the "cakes" and "cookies" were white plastic strips 0.5 cm wide and in lengths of 10, 19 and 27 cm. The entire set of sticks was always freely and conveniently available to the child for all partitioning tasks.

Method

Clinical interaction sessions recorded on video-tape were conducted with each child. Tasks were presented to each child in the following manner.

Task 1

The child was asked to show how he/she would cut the cake so that each person (dolls that were on the table) would have the same amount. The initial question posed was, "How would you cut the cake so that each person has as much?" The number of dolls on the table was first 3, then 5.

Task 2

For the "giant cookie" tasks (triangular and pentagonal), dolls were not placed on the table. Rather, the child was asked, "With how many children would it be easy to share this cookie so that they each have as much?" When the child indicated what number of children he/she could easily partition the cookie for, the child was directed to do the partitioning. The above sequence was repeated to get at the child's second choice for an "easy" partitioning. If the child did not choose either 3 or 5 as an "easy" partition, the child was asked to share the cookie among 3 people if the triangular cookie was on the table, or among 5 people if the pentagonal cookie was on the table.

Sample

For the triangular cookie task, a sample of 31 pupils was drawn from an elementary school in Nova Scotia. Due to unanticipated time constraints for data collection, only 19 of the 31 pupils were requested to do the pentagonal cookie task. Table 1 gives the distribution of the two samples across the grades. The samples were chosen so that there would be a preponderance of higher-grade pupils since on the basis of the previous study, older children would be more likely to have at least one of and perhaps both odd and even partitioning available.

Insert Table 1 About Here

Results

Possible Sequence Effects. In assessing the possible sequence effect present in the earlier study, data from Task 2, in which children were given a free choice to indicate which type of partitioning was easy to do before doing it, is of concern. The data from the 31 children who did the triangular cookie task with free choice is shown in Table 2. The following points are noteworthy.

1. The first choice of 24 of the 31 children was 2 or 4.
2. Only 4 of 31 children chose 3 or 5 first.
3. Rank order of first choices is 2, 4, 3, 5.
4. Even on second choice, only 4 of 31 children chose 3 or 5.
5. The most frequent second choice was 4.
6. The number of successes with 2 and 4 (23) is much greater than for 3 and 5 (0).

Insert Table 2 about here

Data from the 19 children who did the pentagonal task with free choice is shown in Table 3. The pattern of choices, both first and second, and the amount of success experienced with the even and odd cases, is essentially the same as for Table 2.

Insert Table 3 about here

From these two tables is seems overwhelmingly clear that children think it is easier to partition into 2 or 4 parts rather than 3 or 5 parts even when the shapes of the objects to be partitioned are as much or more suited to partitioning into 3 or 5 parts. As well, children experience much more success when partitioning in 2 and 4 parts.

In order to make the above results semantically clear, figure 1 shows all the different types (many occuring more than once) of partitioning schemes of children for the triangular cookie. Of special interest are the partitions for thirds, none of which can be judged particularly correct or suitable given the triangular shape of the object. The dominant use of parallel or orthogonal cuts is striking. It should be remarked that, while parallel or orthogonal cuts do not necessarily give incorrect solutions for odd partitioning problems, what they do provide are approximate solutions based on estimation rather than solutions based on properties of the shapes themselves. The same can be said for fifths. On the other hand, the first example shown for halves was judged correct and suitable, and one that was produced by all but 9 children who chose to partition in 2. It is interesting to note that the vertical cut used in this example was much preferred over any other halving cut. Lesser success was experienced with 4, the only procedure judged correct being indicated by the second entry which was used by 5 children. Again, there is dominant use of parallel or orthogonal cuts, a procedure which many children at this age level seem to apply as a general partitioning strategy without regard for the shape of the object being partitioned.

Insert Figure 1 about here

A listing of all the types of partitions produced by children on the pentagonal cookie is shown in figure 2. A close examination of the schemes for fifths show that, as for thirds, none of the

solutions can be judged particularly correct or suitable given the pentagonal shape of the cookie. Again, the parallel and orthogonal cuts procedures dominate the partitioning shown in figure 2, not only for fifths, but for all partitioning of the pentagon. The partitioning schemes shown in figures 1 and 2 are very much in accord with the partitioning schemes observed in the earlier study which involved circular and rectangular shapes.

Insert figure 2 about here

Role of Shape in Partitioning. Although only two different shapes were used in the study, in both cases children seemed to pay little attention to the constraints of shape, either in deciding what kind of partitioning was easy or in carrying out the "easy" partitioning. Instead of attending to shape, children chose to use schemes involving parallel cuts or orthogonal cuts even though these schemes are not compatible with the shape of the objects being partitioned. With only rare exception, these children had algorithms for partitioning and they used them indiscriminantly for all sharing situations, regardless of the geometrical constraints of the object. This is inspite of the fact that many of the children were nearly finished grade three, and ostensibly had benefitted from some instruction in geometry and fractions over three years.

Conclusions

The study extended, refined and supported the grounded-theory account of the development of partitioning capabilities in children proposed in an earlier study. Support was found for the following conclusions.

1. The possible existence of a sequence effect in the earlier study has been discounted. The dominance of the parallel and orthogonal cuts procedures which work so well for even partitioning, are not artifacts of a mental set induced by asking children to do partitioning for an even number of parts before asking them to share an object among an odd number of people. Indeed, if there was any mental set induced in the present study it would have to be an "odd" set in that Task 1, which immediately preceded Task 2, asked children to partition in 3 and 5. Inspite of this, children overwhelmingly chose to partition in 2 or 4 in the free-choice situation of Task 2. Moreover, children experienced much more success with even partitioning than with odd.

2. The partitioning behaviour of children on triangular and pentagonal shapes is consistent with the theory generated in the earlier study which involved circular and rectangular shapes.

3. Geometrical properties of objects did not play a significant role in the child's partitioning. In the present study, the dominance of parallel and orthogonal cuts as procedures used by children to make sense of any partitioning task regardless of shape constraints also occurred in the earlier study and gave rise to a detailed description of Level II and Level III. In the earlier study, the awareness of geometrical constraints, as for example, using radial cuts instead of parallel or orthogonal cuts on a circular shape, did not emerge until Level IV, and such a conclusion is consistent with the present findings.

4. The child's preferences in choosing what they consider as easy partitioning tasks is consistent with the prediction from the theory that Level III behavior (even partitioning) is prior to Level IV behavior (odd partitioning). Thus, the hierarchical nature of the Levels, based on children's performance, is in accord with the judgement of children as to what sort of partitioning is easy for them. This does not imply that children are in any sense aware of the Levels; it simply implies that the theory and the world as the child makes sense of it are not two completely independent realities.

Discussion

Of interest for on-going study are the findings related to children's judgement of what is easy partitioning. It would seem desirable that any theoretical account of children's construction of mathematics be accompanied by the child's active reflection upon what it is he/she is being asked to do in the situations which give rise to the data. By asking the child to decide what sort of partitioning would be easy to do, the present study engaged the child in consciously reflecting upon the task, and in a minor way participating in what the task was to be before doing it. In this way the gap between the reality of the theory and the reality of the child can be consciously monitored and controlled so that data-driven and theory-driven accounts can find a meaning in the phenomena-driven reality of a child constructing mathematics in his world. More needs to be said on this point since the role of instruction features prominently here, but enough has been said in the present context.

Of particular interest for further study is the finding that for the objects used thus far, awareness of the geometrical properties of the shapes plays little or no role in the partitioning of an object into equal shares. Since various geometrical objects occur often in learning materials used for fraction experiences in the early grades, the findings of the present study question whether there is any valid rationale for including and sequencing these types of activities. For this reason, a systematic examination of children's partitioning behaviour on tasks involving the many geometric shapes often included in early fraction experiences is presently underway.

References

Pothier, Y. and Sawada, D. Partitioning: The emergence of of rational number ideas in young children. Journal for Research in Mathematics Education, 1983, 14, 307 - 317.

TABLE 1

DISTRIBUTIONS OF THE SAMPLES OVER GRADE

Task	Grade			
	K	1	2	3
Triangular Cookie N = 31	5	5	9	12
Pentagonal Cookie N = 19	2	2	7	8

TABLE 2

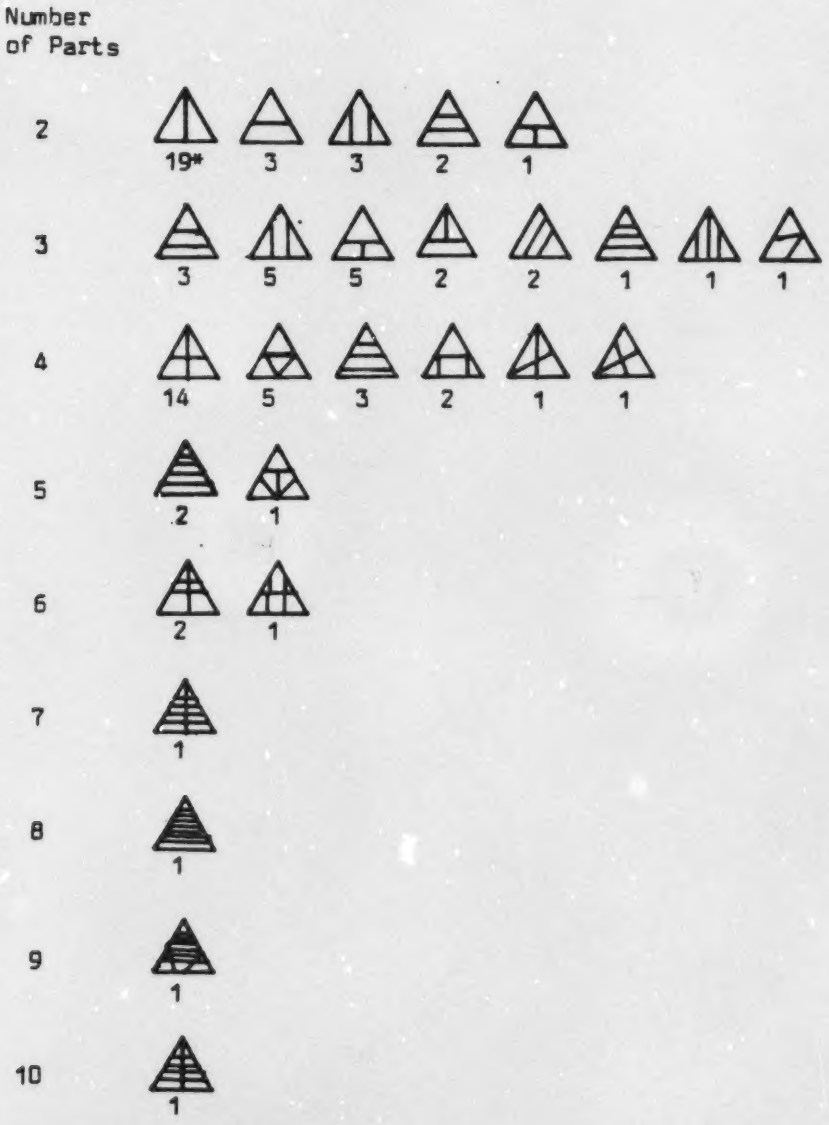
Frequency of Choice of Number of Parts and Success in Partitioning for that Number of Parts for Triangular Cookie

	NUMBER OF PARTS CHOSEN									
	2	3	4	5	6	7	8	9	10	Totals
First Choice	16	3	8	1	1	1	0	1	0	31
Second Choice	2	2	10	2	3	0	1	0	1	21
Total Choices	18	5	18	3	4	1	1	1	1	52
Successful Partitioning	18	0	5	0	0	0	0	0	0	

TABLE 3

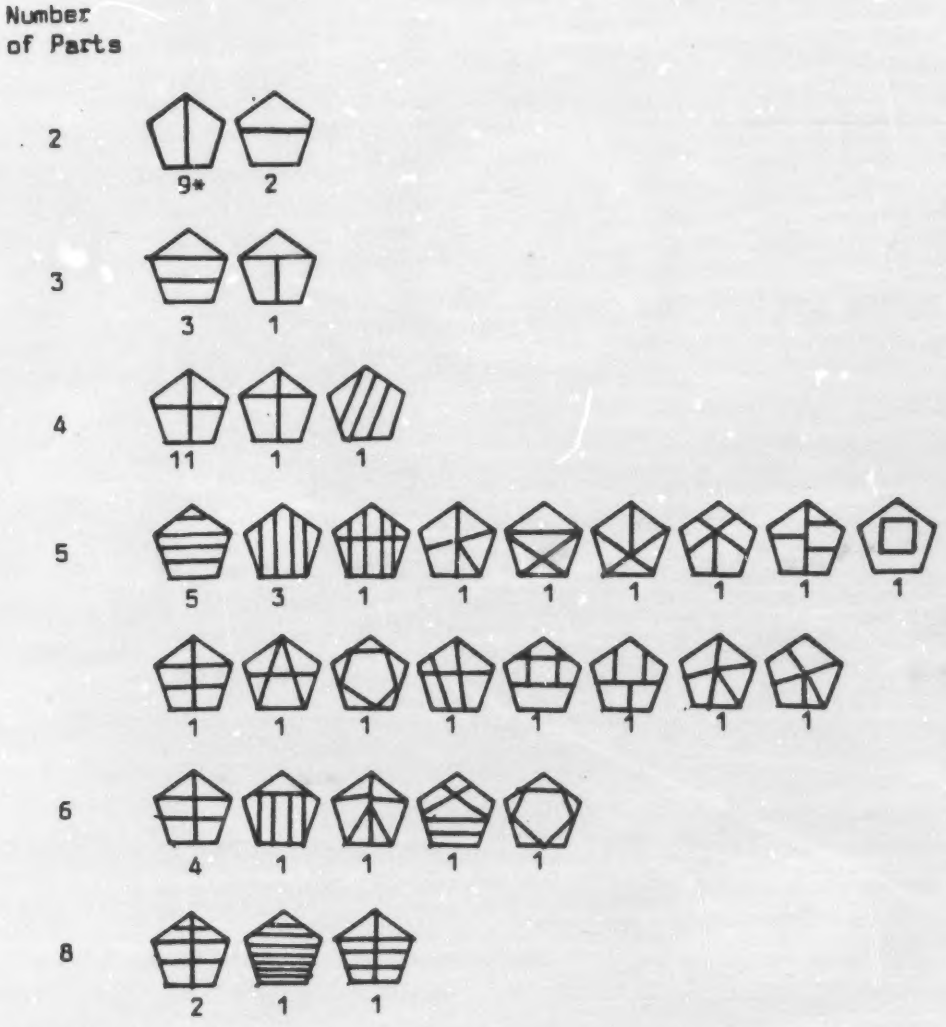
Frequency of Choice of Number of Parts and Success in Partitioning for that Number of Parts on Pentagonal Cookie

	NUMBER OF PARTS CHOSEN								
	2	3	4	5	6	7	8	9	Totals
First Choice	7	1	5	2	3	0	1	0	19
Second Choice	2	3	8	1	2	0	1	0	17
Total Choices	9	4	13	3	5	0	2	0	36
Successful Partitioning	9	0	0	0	0	0	0	0	



*Indicates frequency of use.

Figure 1. Triangular shape: Partitioning schemes listed in order of frequency of use within each row.



*Indicates frequency of use.

Figure 2. Pentagonal shape: Partitioning schemes listed in order of frequency of use within each row.